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14. ABSTRACT

Satellite attitude controllers have been designed for a rigid spacecraft whose only actuators are magnetic torque rods. This effort's goals have been to develop a new class of light-weight, moderate-accuracy attitude controllers and to evaluate and further develop general methods for the control of time-varying systems. Three different classes of controllers have been developed and simulation tested, one based on linear quadratic regulator techniques, one based on sliding-mode-like concepts, and one based on new H-infinity techniques for time-varying systems. These H-infinity controllers achieve the best performance. In addition to the controller design studies, the issue of attenuation of constant 3-axis disturbances has been addressed. Disturbance attenuation is difficult for this system because it can apply torques only about the 2 axes that are perpendicular to the Earth's magnetic field. It is a challenge to determine how best to counteract a low-frequency 3-axis disturbance torque, on average, via judicious use of the fact that the Earth's magnetic field direction varies in time as the spacecraft moves along its orbit. Pointing accuracies on the order of 1 deg or better have been demonstrated in the presence of typical levels of disturbance torque.

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Satellite Attitude Control Using Magnetic Torquers, a Periodic Time-Varying Control Problem

Final Technical Report for AFOSR GRANT F49620-99-1-0169

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Abstract

Satellite attitude controllers have been designed for a rigid spacecraft whose only actuators are magnetic torque rods. This effort's goals have been to develop a new class of light-weight, moderate-accuracy attitude controllers and to evaluate and further develop general methods for the control of time-varying systems. Three different classes of controllers have been developed and simulation tested, one based on linear quadratic regulator techniques, one based on sliding-mode-like concepts, and one based on new H-infinity techniques for time-varying systems. These H-infinity controllers achieve the best performance. In addition to the controller design studies, the issue of attenuation of constant 3-axis disturbances has been addressed. Disturbance attenuation is difficult for this system because it can apply torques only about the 2 axes that are perpendicular to the Earth's magnetic field. It is a challenge to determine how best to counteract a low-frequency 3-axis disturbance torque, on average, via judicious use of the fact that the Earth's magnetic field direction varies in time as the spacecraft moves along its orbit. Pointing accuracies on the order of 1 deg or better have been demonstrated in the presence of typical levels of disturbance torque.

1.0 Introduction

1.1 Goals of Research

This research effort had two related goals. One was to design improved feedback controllers for doing spacecraft attitude control using only magnetic torque rods as actuators. The other was to apply and further develop controller design methods for periodically time-varying systems.

The rigid-body attitude dynamics for a nadir-pointing, Earth-orbiting spacecraft can be approximated by a periodic system model if the actuation is provided solely by magnetic torque rods. It is a challenging problem to a design robust controller for such a system because the system is under-actuated – the torque rods can only apply torques perpendicular to the local Earth's magnetic field – and because models of the Earth's magnetic field can have significant errors. New methods have been developed that should be able to accomplish this task. One of these methods relies on H-infinity-type techniques for optimization of input-output gains of systems, and it employs a generalization to periodic systems of the concept of an input-output gain.

The plan of the project has been to test these new design techniques on a practical problem of significant technical importance: the magnetic attitude control problem. The tests have served two purposes. One has been to demonstrate the strengths and weaknesses of the proposed controller design techniques. The other has been to develop a new generation of active magnetic attitude controllers that could be used on small satellites to achieve better pointing performance than had previously been possible with a purely magnetic-torquer-based system.

1.2 Relationship of Research to Needs of Air Force and Department of Defense Programs

These results are relevant to the Air Force's mission for two reasons. First, they advance the state of the art for attitude control of spacecraft. The attitude control systems that can be designed are practical for use on small spacecraft because they do not employ heavy gravity gradient booms or wheels. Such controllers would be ideal for a constellation of small, simple, light-weight communications spacecraft.

Pointing accuracies on the order of 0.5 deg have been demonstrated via simulation testing. This is considerably better than that available from current gravity gradient systems. This is especially significant because these accuracies are achievable in the presence of significant 3-axis disturbance torques and system modeling error. Analyses have shown that, with proper spacecraft design, it may be possible to achieve pointing accuracies on the order of 0.1 deg. This level of performance would start to compete with coarser wheel-based systems, yet at a fraction of the weight and with more reliability.

The second important application area for the results of this research is in the control of time-varying systems, especially periodic systems. This research demonstrates the usefulness of H-infinity techniques to a significant time-varying problem, and it develops another new general technique for periodically time-varying systems. Other periodic systems to which these concepts can be applied include receiver demodulation circuits, helicopter blade flexure, and robotic systems in repetitive motion (e.g., walking robots). The more general concepts of H-infinity time-varying control can be applied to any highly maneuvering vehicle that has nonlinear dynamics.

1.3 Summary of this Project's Efforts and Accomplishments

A number of different efforts have been carried out as part of this project. One part of the project has been to develop suitable models for controller design and simulation testing. Two basic system models and simulations have been developed. One is a simple linear time-varying model for use in controller design and analysis. The other is a much more complex model for use in hi-fidelity simulation testing of candidate designs. It includes attitude nonlinearities, orbit perturbations, Earth rotation effects, and higher harmonics of the Earth's magnetic field.

Another effort considered the issue of quasi 3-axis disturbance attenuation. The system cannot totally eliminate the effects of a general 3-axis disturbance torque because magnetic torque can be applied only about 2 axes. Time-variation of the unactuated axis allows for attenuation of the average effects of a general 3-axis disturbance torque, but at the cost of periodic oscillations. This part of the study considered the relationship between the resulting induced oscillations and the orbit, the applied disturbance torque, and the spacecraft's inertial characteristics.

Three different types of controllers have been designed, analyzed, and simulation tested for this system. One is a periodic linear-quadratic regulator. Work has been done to develop a simple form of this time-varying controller and to adapt it to deal with actuator saturation. This controller has been considered because of its simple form and because it is able to incorporate sensor information that enhances robustness. The second type of controller that has been considered is one that resembles a sliding mode controller. It has been considered because it also can achieve robustness through judicious use of available sensor data. The third controller is the one based on H-infinity techniques. Two different controllers of this class have been designed

and tested, and their performance has been compared with that of the periodic linear quadratic regulator.

1.4 Outline of Report

The remainder of this report consists of 3 sections. Section 2 lists the personnel involved in the project and the publications that have resulted from this work. Section 3 presents significant results from the different studies that have been carried out as part of this project. Section 4 is a summary of the report.

2.0 Personnel and Publications

Three different researchers have been involved in this project: Maria Hagan, a master of science student, and Profs. Raffaelo D'Andrea and Mark Psiaki. All three researchers were part of Cornell's Sibley School of Mechanical and Aerospace Engineering during all or part of this project. Maria Hagan did her masters thesis on optimal 3-axis disturbance attenuation using magnetic torquers. Prof. D'Andrea worked on H-infinity-based design and analysis of magnetic-torquer-based attitude controllers. Prof. Psiaki worked on modeling and simulation and on the design and analysis of the other two types of controllers that have been considered: periodic linear quadratic regulators and sliding-mode-like controllers.

Two publications have resulted from this work, and a third is planned. The first publication was Maria Hagan's masters thesis, which is entitled "Satellite Attitude Control for Steady State Disturbance Using Only Magnetic Torquers" [1]. The second publication is a paper by Prof. Psiaki on an asymptotic periodic linear quadratic regulator for magnetic attitude control [2]. The planned third publication will be about the application of H-infinity techniques to the problem.

3.0 Research Results

3.1 Achievable 3-axis Disturbance Attenuation via Quasi 3-Axis Magnetic Torquer Control

The issue of how to attenuate the effects of constant or low-frequency 3-axis disturbance torques has been studied extensively, and the results are reported in Ref. 1. The following section is a summary of that reference's results.

In the case of a nadir-pointing spacecraft, one of the main functions of an attitude control system is to counteract the effects of small disturbance torques. Many of these disturbances are constant or have a significant steady-state component. Examples of such disturbances are atmospheric drag torques and gravity gradient torques due to misalignment of the principal axes.

Although the system is theoretically controllable, it is difficult to counteract disturbance torques using only magnetic torquers. Magnetic torquers produce a controllable spacecraft dipole moment vector, m. This vector gets crossed into the Earth's magnetic field, b, to produce the control torque, $n_c = m \times b$. Therefore, n_c is always perpendicular to b. A disturbance torque, n_d, will normally have a component that is parallel to b. This component cannot be counteracted instantaneously.

If the spacecraft's orbit is inclined, then the average effects of steady-state disturbances can be counteracted using magnetic actuation. This is true because the Earth's magnetic field moves with respect to the roll/pitch/yaw reference axes as the spacecraft moves about its orbit. Over time, a magnetic controller can produce a net average torque about 3 axes.

Unfortunately, the controller also excites periodic oscillations of the satellite as a by-product of its production of an average 3-axis torque. This is illustrated in Fig. 1. The figure shows two steady-state yaw response time histories for a satellite that is experiencing constant

pitch and yaw disturbance torques. The dashed horizontal line at 8.6° is the open-loop yaw response, and the solid curve is the response for a particular magnetic controller. The controller successfully eliminates the steady-state bias in the response, but it also excites a periodic response with a periodicity equal to the orbital period. This periodic response is a direct result of using a time-varying 2-axis control to cancel, on average, a 3-axis disturbance.

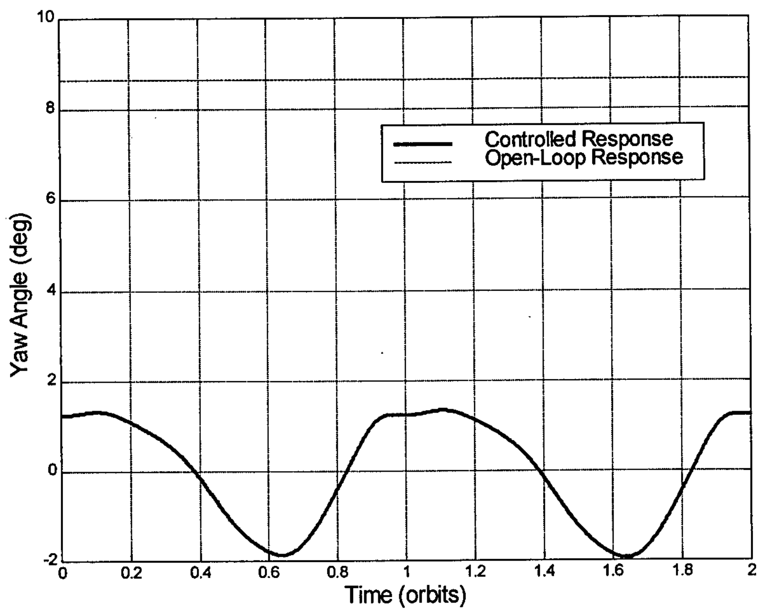

Fig. 1. Yaw angle responses to steady pitch and yaw disturbance torques, 75° inclination.

The magnitude of the induced periodic response depends on many factors. It is a function of the controller design, the orbital inclination, the spacecraft's mass moment-of-inertia properties, and the magnitude and direction of the constant disturbance torque. The periodic response also introduces coupling. For example, when counteracting a yaw disturbance, there will be a periodic component to the roll response. In some cases, the resulting oscillations may be larger than the steady-state response that would occur without control, or even if smaller, the periodic controlled response may be less acceptable to the overall mission of the spacecraft.

A systematic study has been made of the periodic response that occurs during 3-axis disturbance attenuation via magnetic torquing. For a given orbit and spacecraft inertia matrix, the control that produces the minimum root-mean-square response has been computed. The magnitude of this minimum response has been studied as a function of the orbital inclination, the inertia parameters, and the disturbance torque's direction.

If the orbital inclination is high and if the spacecraft inertia parameters are chosen properly, then magnetic torquers can produce much lower responses to steady-state disturbances than can passive gravity-gradient systems. This is especially true if the dominant disturbance torque is a pitch torque, which is often the case in low Earth orbit. The best inertia parameters are in regions of configuration space that would make the open-loop system unstable with respect to gravity gradient torques. Open-loop instability might be acceptable because, as will be discussed in the next sections, magnetic-torque-based controllers can achieve robust 3-axis attitude stabilization.

At lower orbital inclinations, it is a bad idea to use magnetic torquers to try to counteract steady 3-axis disturbance torques. The peaks of the resulting oscillations tend to be as large or larger than the steady-state response that can be achieved with a gravity-gradient stabilized system. Therefore, at lower inclinations it might be wise to use magnetic control only for stabilization. The open-loop gravity-gradient properties of the spacecraft could be used to counteract the disturbance torques, which would imply that some attitude bias would need to be accepted by the designers.

A systematic comparison has been made between the 3-axis disturbance attenuation capabilities of two systems, an optimal magnetic-torquer based system and an open-loop gravity-gradient stabilization system. The results of this study are summarized by Fig. 2. This figure shows a weighted sum of maximal attitude responses to disturbances. This weighted sum is a

function of the spacecraft's inertia ratios, σ_x = (pitch inertia – yaw inertia)/(roll inertia) and σ_z = (pitch inertia – roll inertia)/(yaw inertia). In order to normalize these plots, the absolute inertias have been chosen to maintain a constant sum of the 3 principal inertias for each different spacecraft configuration. Each of the four plots in the figure uses contour shading to plot the weighted maximal attitude response as a function of σ_x and σ_z. The contour shades are mapped to weighted attitude response levels via the shade bar that appears to the right of each plot in the figure. The weighted attitude response is constructed from the "gains" from constant disturbance input torques around each satellite axis to the resulting steady-state peak attitude response on each axis. The upper left-hand plot shows the response of an open-loop gravity-gradient system. This plot is valid only in the two regions that are not crossed by diagonal lines because these are the only stable regions in the 2-dimensional inertial parameter space of the figure [3]. All regions of parameter space show contour values for the other three plots because active magnetic torquer control can be used to stabilize any spacecraft configuration. The upper right-hand graph is for magnetic disturbance suppression at an orbital inclination of i_m = 90 deg from the magnetic equation, and the two bottom plots are for magnetic inclinations of 78 deg and 67 deg.

Figure 2 shows that the best closed-loop 3-axis disturbance rejection performance is better than the best open-loop performance. It is more than 6 times better in terms of the weighted gains from disturbance torque to worst-case attitude excursion. The plots also show that the best performance for the magnetically controlled system occurs at a different inertial configuration than that which gives the best open-loop performance. The best open-loop configuration is in the neighborhood of $\sigma_x = \sigma_z = 1$, and the best closed-loop configuration is near $\sigma_x = \sigma_z = -1$. Thus, the best open-loop configuration is a long thin structure with its longest axis pointed towards the

Earth and its intermediate-length axis pointed along the velocity vector; this is a traditional gravity-gradient stabilized configuration. The best closed-loop design is a long thin structure with its longest dimension oriented along the orbit normal.

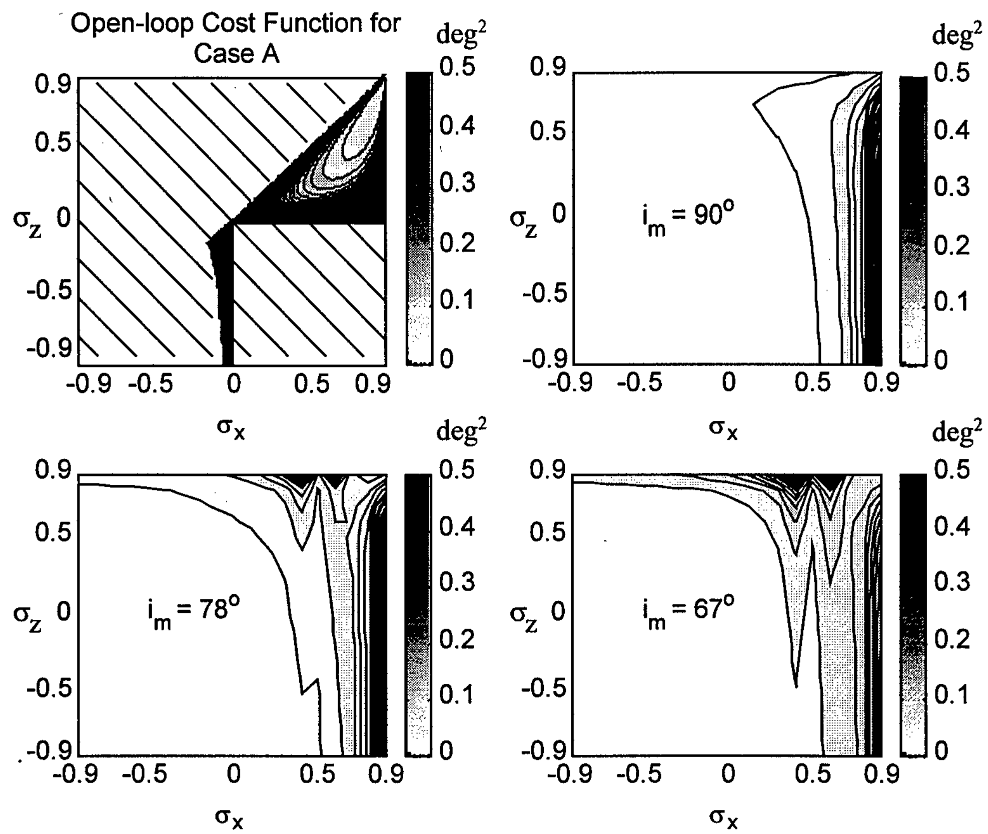

Fig. 2. Weighted 3-axis maximal attitude response to disturbances, a comparison between open-loop gravity gradient performance (upper-left-hand plot) and magnetic attitude control at 3 different orbital inclinations (other three plots).

3.2 Asymptotic Periodic Linear Quadratic Regulation and Its Application to Magnetic Satellite Attitude Control

Reference 2 describes a new asymptotic low-bandwidth linear quadratic regulator for a periodic system, and it applies this regulator to the magnetic attitude control problem. This section summarizes Ref. 2's results.

The asymptotic linear quadratic regulator theory can be used to design controllers for time-varying periodic linear systems whose models take the form:

$$\dot{x} = A\,x + B(t)\,u \qquad (1)$$

where x is the system state vector, A is the constant open-loop dynamics matrix, and $B(t)$ is the periodic control effectiveness matrix: $B(t) = B(t+T)$, with T the period of the system. The magnetic attitude control problem has a linearized system model in this form where x contains the attitude and rate perturbations from the nadir-pointing configuration, and u contains the dipole moments of the magnetic torque rods.

The asymptotic linear quadratic control laws that have been designed for this system take the following form

$$u_{nom} = -\,\alpha_0\, R^{-1}B^T(t)P_{ss}\,x \qquad (2a)$$

$$\beta = \max_i \frac{|(u_{nom})_i|}{(u_{max})_i} \qquad (2b)$$

$$u = \begin{cases} u_{nom} & \text{if } \beta \le 1 \\ \frac{1}{\beta}u_{nom} & \text{if } 1 < \beta \end{cases} \qquad (2c)$$

where u_{nom} is the nominal control vector when there is no control saturation, and β is a scalar that controls the action of the saturation logic when any element of u_{nom} exceeds the corresponding actuator saturation limit in u_{max}. The matrix R is the control weighting matrix in the linear quadratic control problem, and the matrix P_{ss} is an approximate solution of the periodic-Riccati equation. P_{ss} is constant. The true steady-state solution of the Riccati equation is periodically time-varying, but Ref. 2 proves that it approaches P_{ss} asymptotically in the limit as R approaches infinity. Thus, for $\alpha_0 = 1$, the control law in eq. (2a) approaches the solution to the periodic linear quadratic regulator problem in the limit of large control weighting. The large

control weighting limit is also known as the low-bandwidth limit. Note that this controller design assumes full state feedback, as do all controller designs in this report. This is a reasonable assumption for many satellite applications.

The scalar $\alpha_0 >> 1$ is part of the control saturation logic. It allows the designer to use a very large R. This yields a nominal controller that would produce a very slow response and avoid control saturation for large state errors. This is the system's effective response when control saturation occurs that causes the ratio α_0/β to be near 1. When the state error is small, a large value of α_0 allows the controller to recover bandwidth because α_0 scales up the gain. In this regime, the controller makes use of the infinite gain margin of a linear quadratic regulator. This method of dealing with control saturation is adapted from Ref. 4, where it used for time-invariant systems. This is the first known application to time-varying systems.

The form of eq. (2a) is very important in the magnetic torquer attitude control problem. The time varying gain of the controller is $\alpha_0 R^{-1}B^T(t)P_{ss}$, and the only time-varying component of this gain is $B(t)$, the control effectiveness matrix. In a real spacecraft, this matrix can be computed from magnetometer measurements and a model of the spacecraft's inertial properties. This means that the $B(t)$ matrix used in the actual controller will have very little modeling error, a fact which helps the system to achieve robustness.

The control-law form in eq. (2a) also solves the difficult problem of controller synchronization: If the control law had been of the usual periodic form: $u = -K(t)x$ with $K(t) = K(t+T)$, then there would have been a need to synchronize with the actual time-variations of the system, which are not exactly periodic due to Earth rotation and orbit precession.

A third benefit of the control law in eq. (2a) is that it does not entail the storage of matrix time histories such as a $K(t)$ time history. It only entails storage of the constant matrices R and P_{ss} and the constant scalar α_0.

Another important contribution of the linear quadratic regulator design is that it can incorporate integrators. These can be used to counteract the average effects of steady-state disturbances, along the lines of what was studied in Ref. 1. Integrators are added via state augmentation: One appends 3 integrator states to the model in eq. (1), and these are fed back in the control law of eqs. (2a)-(2c). Although such disturbance attenuation techniques are not optimal, they have proved effective in combating the effects of typical disturbance torques.

Asymptotic linear quadratic regulator control laws have been tested in a variety of situations. Their robustness to modeling uncertainty has been evaluated by probing the effects on Floquet stability of parametric differences between the design model and the actual system. Stability is maintained over a very large range of parameter uncertainty. Transient and steady-state performance have been explored using linear and nonlinear simulations.

As an example, the attitude and control input time histories for a typical case are shown in Figs 3. and 4. These figures show that the system converges from initial attitude errors of 30 deg on all three axis. This is remarkable for two reasons: because the controller has been designed using a linearized attitude dynamics model and because control saturation occurs – see the start of Fig. 4. Figure 3 also shows that the steady-state attitude errors are small; a closer look at the data reveals that the peak steady-state errors are 0.7 deg for roll and pitch and 1 deg for yaw. These errors are caused by small disturbance torques. The closed-loop yaw response is a great improvement over the open-loop response, and the roll/pitch response is similar to that of the

open-loop system; the corresponding open-loop steady-state roll, pitch, and yaw errors are 0 deg, 0.5 deg, and 7.8 deg, respectively.

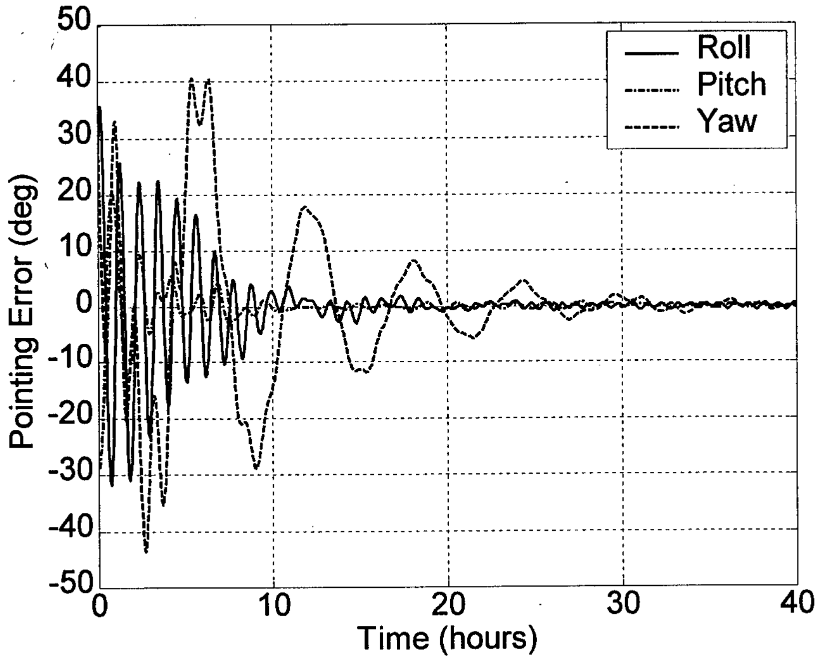

Fig. 3. Pointing error time histories, a typical asymptotic linear quadratic regulator case.

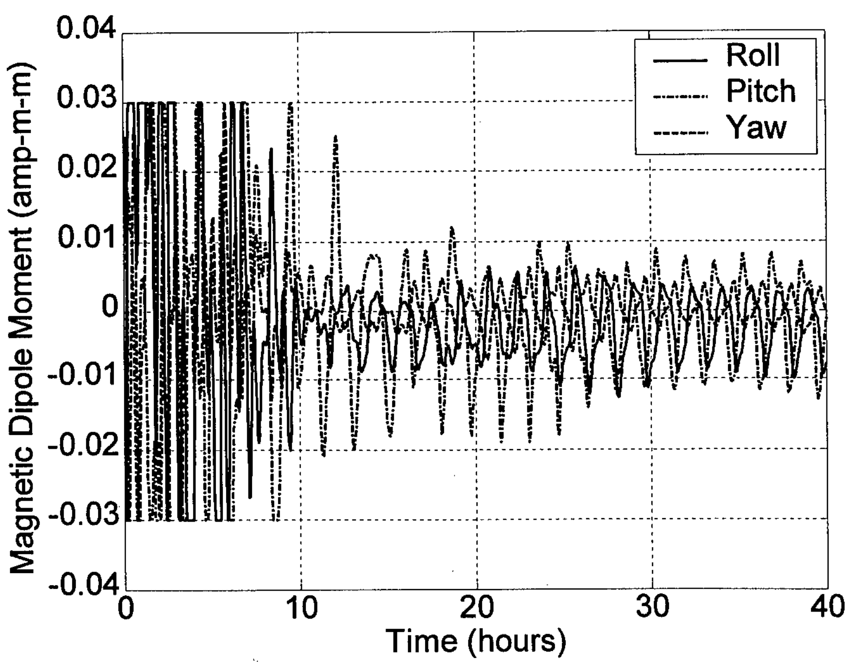

Fig. 4. Control input time histories, a typical asymptotic linear quadratic regulator case.

3.3 Sliding-Mode-Like Magnetic Attitude Controller Design

An important direction in the spacecraft coordinate system is the direction about which angular accelerations cannot be instantaneously controlled. It is $\hat{v} = I_{sc}b/||I_{sc}b||$. This direction will be skewed from the untorqueable direction b if the spacecraft inertia matrix, I_{sc}, does not have equal principal inertias, which is normally the case. A fast/slow controller design approach can be implemented by making careful use of the vector $\hat{v}$. This approach is related to sliding mode control. It splits the controller design problem into two subspaces, one perpendicular to $\hat{v}$ and the other parallel to $\hat{v}$. In the subspace perpendicular to $\hat{v}$, the system can be controlled on very short time scales, but control parallel to $\hat{v}$ requires long time scales and use of interactions between the two subspaces. This approach has been inspired by the work of Ref. 5.

The controller in the subspace perpendicular to $\hat{v}$ is a fast controller. It is a simple decoupled proportional-derivative controller that uses feedback linearization principles in order to approximately achieve a desired rate of angular acceleration in this subspace. This fast part of the controller accepts attitude commands for the fast states and tries to track these attitude commands in the subspace perpendicular to $\hat{v}$. These attitude commands come from the slow part of the controller. Suppose that the attitude commands are represented by the vector r_{fast}. Then the fast part of the controller takes the form:

$$\dot{\omega}_{des} = k_p\left(r_{fast} - \begin{bmatrix}\phi\\ \theta\\ \psi\end{bmatrix}\right) + k_d\left(\dot{r}_{fast} - \begin{bmatrix}\dot{\phi}\\ \dot{\theta}\\ \dot{\psi}\end{bmatrix}\right) - k_d\,\dot{\hat{v}}\hat{v}^T\left(r_{fast} - \begin{bmatrix}\phi\\ \theta\\ \psi\end{bmatrix}\right) \tag{3a}$$

$$n_{des} = I_{sc}\,(I - \hat{v}\hat{v}^T)(\dot{\omega}_{des} - \dot{\omega}_{ol}) \tag{3b}$$

$$u = \frac{b \times n_{des}}{b^T b} \tag{3c}$$

In this control law $\dot{\omega}_{des}$ is the desired angular acceleration, which is designed to cause the attitude to line up with r_{fast} in the subspace perpendicular to $\hat{v}$, k_p and k_d are constant scalar proportional and derivative feedback gains, ϕ, θ, and ψ are, respectively, the roll, pitch, and yaw attitude errors, n_{des} is the torque that the controller would like to apply using the magnetic torque rods, and $\dot{\omega}_{ol}$ is what the angular acceleration would be if there were no magnetic control torque. The inclusion of $\dot{\omega}_{ol}$ in eq. (3b) is a sort of plant inversion.

The slow part of the controller deals with the projections of the attitude and rate errors onto the direction $\hat{v}$. It feeds these errors back, through dynamic compensation, to create the attitude command for the fast part of the controller, r_{fast}. Given appropriately defined command directions perpendicular to $\hat{v}$, it is possible to show that the slow system can be modeled approximately by a time-invariant linear system. Therefore, time-invariant control design techniques can be used to design the slow controller. The only caveat for the slow controller design is that it must have closed-loop response times that are slow compared to those of the fast part of the closed-loop system. The slow part of the control law takes the form:

$$\dot{z} = A_{slow} z - K_{slow} \begin{bmatrix} \hat{v}^T \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix} \\ \hat{v}^T \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \end{bmatrix} \tag{4a}$$

$$\mu = \hat{v} \times \hat{p} \tag{4b}$$

$$\eta = (I - \hat{v}\hat{v}^T)\hat{p} \tag{4c}$$

$$r_{fast} = \mu z_1 + \eta z_2 \tag{4d}$$

In this control law, z is a vector of 2 compensator states for the slow part of the controller: $z = [z_1;z_2]$, A_{slow} and K_{slow} are constant 2×2 controller design matrices for the slow part of the controller, $\hat{p}$ is the unit vector along the pitch axis, and μ and η are two direction vectors that are used to define the commanded attitude in the fast subspace.

One can think of r_{fast} as the "control input" to the slow system. The use of μ and η to construct r_{fast} in eq. (4d) allows a time-invariant slow controller design. The vector $\hat{v}$ tends to rotate about the pitch axis for a nadir-pointing spacecraft in an inclined orbit. Therefore, the vector μ also tends to rotate about the pitch axis, and η stays roughly aligned with the pitch axis. The possibility of designing A_{slow} and K_{slow} using time-invariant techniques is a direct result of the fixed relationships of μ and η to $\hat{v}$.

Even though it is designed using time-invariant techniques, the fast/slow controller is, effectively, a time-varying controller. This is so because its control law makes use of the $\hat{v}$ vector, which is a time-varying vector in spacecraft coordinates. For a nadir-pointing spacecraft, $\hat{v}$ rotates with respect to spacecraft coordinates at a frequency of once per orbit.

The fast/slow controller is robust with respect to various parameter perturbations. The robustness of the design comes from the fact that $\hat{v}$ can be accurately known at any given time. It is a function of the field vector, b, which is a measured quantity, and of the spacecraft inertia matrix, I_{sc}. Even though there is some uncertainty in I_{sc}, the resulting uncertainty in $\hat{v}$ is not significant enough to cause serious performance degradation.

Stability robustness has been evaluated by a Floquet analysis procedure. The *1*-orbit state transition matrix has been calculated for a variety of mismatches between parameters in the controller design model and parameters in the "truth" model. In all cases considered, the

eigenvalues of the state transition matrix stay well within the unit circle, which demonstrates stability robustness.

As an example, Fig. 5 shows the transient response of a spacecraft that is controlled magnetically using the fast/slow controller. The plot shows good convergence of the roll, pitch, and yaw angle time histories despite significant levels of mismatch between the system model that was assumed in the controller design and the "truth" model that was used in the simulation. The simulation "truth" model had an orbital inclination of *75*°, an altitude of *700* km, and roll, pitch, and yaw principal inertias of *12.3* kg-m^2, *13.0* kg-m^2, and *6.8* kg-m^2, respectively. The controller design, on the other hand, assumed an inclination of *90*°, an altitude of *600* km, and principal inertias of *8.7* kg-m^2, *10.0* kg-m^2, and *6.5* kg-m^2. Also, the controller design assumed a dipole model for the Earth's magnetic field, but the simulation used terms that mimic the Earth's field out to the 6th harmonic. Clearly, the controller shows no ill effects from these parameter variations.

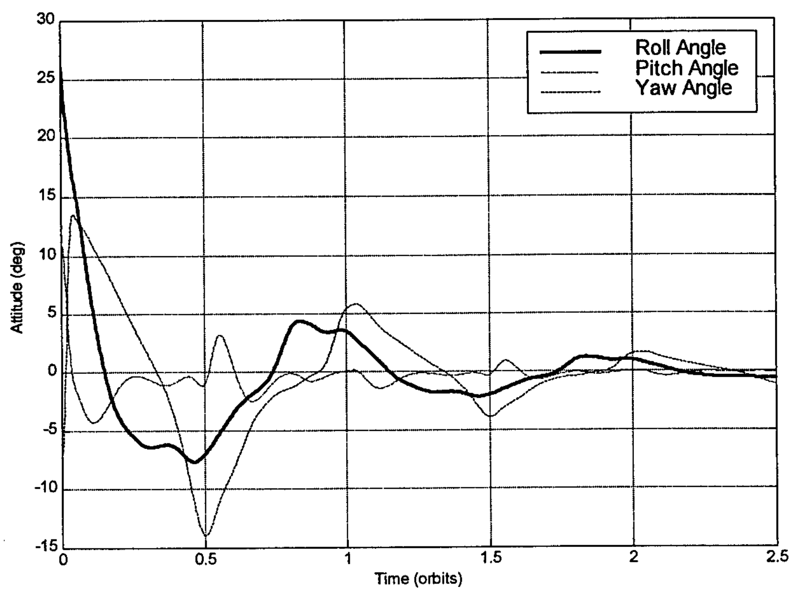

Fig. 5. Transient attitude response of fast/slow controller when there are parameter perturbations from the design model.

3.4 Magnetic-Torquer-Based Attitude Controller Design that Uses H-Infinity Techniques for Periodically Time-Varying Systems

The third attitude control design strategy was to use optimal disturbance rejection control design for linear time-varying systems. As described in Section 3.2, the first order satellite dynamics can be captured by the following time varying system:

$$\dot{x}(t) = Ax(t) + B_1 d(t) + B_2(t)u(t) \tag{5a}$$
$$z(t) = C_z x(t) + D_{zu} u(t) \tag{5b}$$
$$y(t) = C_y x(t) + D_y d(t) \tag{5c}$$

In the above equations, $x(t)$ is the state of the system at time t, d is a vector of disturbance variables whose effects must be attenuated, z is a vector of error variables which must be kept small, y is a vector that contains the sensor variables available to the controller, and u is the control input vector that is generated by the feedback law. These equations of motion were discretized over one orbit by assuming constant inputs during a sampling period T_S. This yields a periodic, discrete-time, linear time-varying system of the following form:

$$x(T_S(k+1)) = \bar{A}x(T_S k) + \bar{B}_1 d(T_S k) + \bar{B}_2(T_S k)u(T_S k) \tag{6a}$$
$$z(T_S k) = \bar{C}_z x(T_S k) + \bar{D}_{zu} u(T_S k) \tag{6b}$$
$$y(T_S k) = \bar{C}_y x(T_S k) + \bar{D}_y d(T_S k) \tag{6c}$$

The tools described in Ref. 6 were then used to design a periodic, discrete-time, linear time-varying controller for the above plant equations. It was designed to minimize the energy-to-energy gain of the closed loop system that is depicted in Fig. 6. The disturbance vector d was taken to be a constant torque of 0.5×10^{-6} N-m about each spacecraft axis. The error vector z was formed as a composite of x and u. The sensors available for control were the full state of the

system x and the disturbances d. The control variables u were the magnetic torque rod dipole moments, as described in Section 3.1.

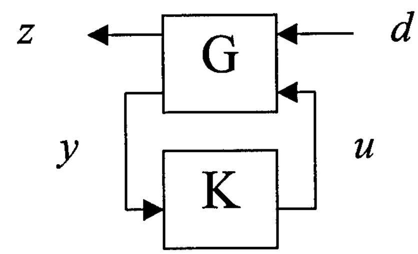

Fig. 6 Closed-loop system block diagram for a time-varying controller design based on energy gain minimization.

Two different control designs were performed. In the first, a frequency-independent unit weighting was used for the disturbance vector d. In the second design, the disturbances were weighted by an integrator in order to reflect their constant nature. Results for the second controller may be found in Figs. 7 and 8. In Fig. 7, the angular perturbations are plotted vs. time (measured in orbits), while Fig. 8 plots of the corresponding control effort. For purposes of comparison, Fig. 7 also plots the angular perturbations for an asymptotic low-bandwidth linear-quadratic regulator operating on the same system and with the same disturbances; this latter controller is of the form described in Section 3.2. As can be seen from Fig. 7, the energy-gain-minimizing controller with integrators does better than the linear-quadratic regulator in two respects: it settles faster, and its steady-state roll responses to disturbance is significantly lower. Its steady-state yaw response is slightly worse than that of the linear quadratic regulator, but this is not significant because yaw is not the largest attitude perturbation. Both controllers' responses

are much better than the open-loop equilibrium response, which is 1.2 deg in roll, 3.9 deg in pitch, and 17.7 deg in yaw.

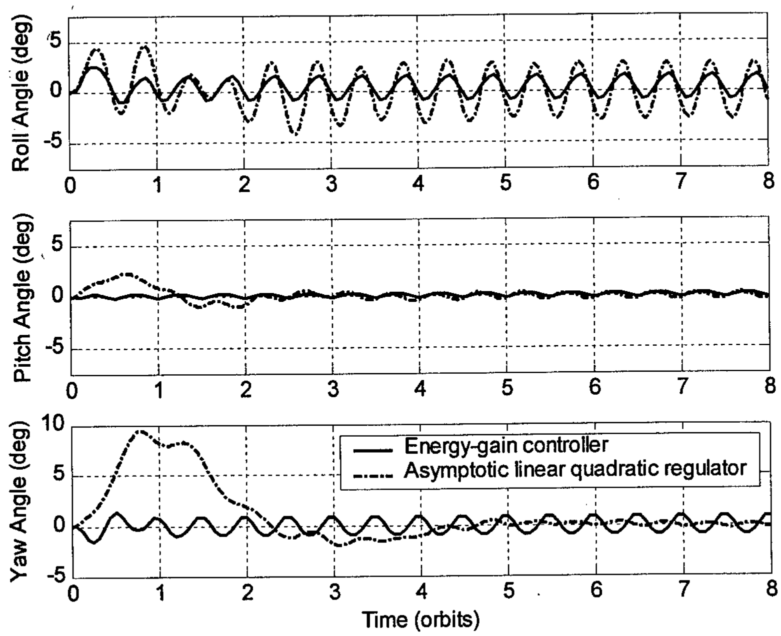

Fig. 7. Angular error time history comparison between energy-gain-minimizing controller with integrators and asymptotic linear quadratic regulator with integrators.

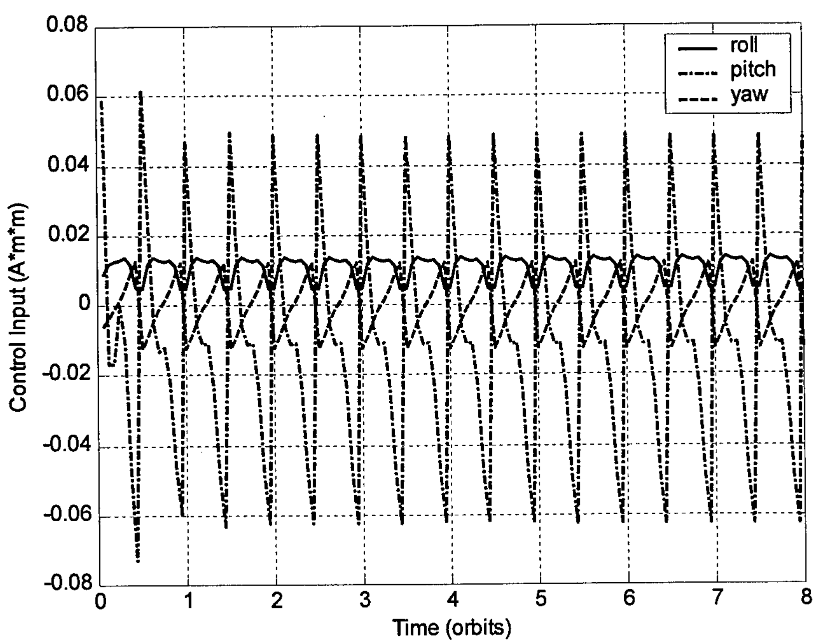

Fig. 8. Control input time histories for an energy-gain-minimizing controller with integrators.

4.0 Summary

The problem of satellite attitude control via magnetic actuation has been used as a means of testing and further developing methods for controlling periodically time-varying systems. Magnetic-torque-rod-based attitude control is difficult because the system is inherently under actuated and time-varying. In order to achieve good performance, a controller must exploit model information about the future directions in which the Earth's magnetic field will point. This information is subject to modeling error. Therefore, controller robustness is critical to the success of such a system.

Four controller studies have been carried out. The first was an evaluation of the ability to counteract a general 3-axis disturbance torque with an under-actuated system, one that can only torque about 2 axes at any given time. Rotation of the untorqueable axis allows steady-state 3-axis disturbance rejection on average, but at the cost of induced oscillations. The second study developed a new asymptotic low-bandwidth periodic linear quadratic regulator that has a simplified control law form. This control law form is ideally suited to the magnetic attitude control problem because it makes good use of available measurements of the Earth's magnetic field to achieve system robustness. This second study also dealt with the issue of actuator saturation for a periodic feedback controller. The third study developed a sliding-mode-like controller that decomposes the state space into two subspaces, one along which angular acceleration can be affected instantaneously and one whose angular acceleration is not directly affected by the control input. This decomposition allows for decoupled fast/slow controller design, and it is robust because the time-varying state space decomposition can be derived

primarily from sensor data. The fourth study developed periodic controllers for the system that use H-infinity techniques to minimize generalized gains for the time-varying system.

The H-infinity-based design technique has achieved better performance than the other design techniques that have been considered in this study. The best H-infinity-based controller has a faster settling time and smaller steady-state responses to disturbances. Unlike the asymptotic periodic linear quadratic regulator and the fast/slow controller, however, the H-infinity controllers do not use magnetometer measurement data. Therefore, they do not achieve the levels of robustness with respect to model uncertainty that they could achieve if they were to use this data.

In addition to its work on the magnetic attitude control problem, this study has made two contributions to the general area of control of periodic systems. One contribution is its new asymptotic periodic linear quadratic regulator. The asymptotic regulator can be simpler to implement than a general time-varying periodic controller because many of its matrices are constant. Also, this controller can be adapted to include integral control and saturation logic. The other general contribution of this work is its demonstration that H-infinity gain minimization techniques can be applied to a practical time-varying control problem. The best H-infinity design exhibited superior performance. This demonstrates that more attention should be given to the problem of applying H-infinity concepts to time-varying problems.

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